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System and Disturbance Identification for Feedforward and Feedback Control Applications

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This paper examines the problem of system identification in the presence of unknown disturbances. Specifically, we consider the case where the disturbance effect on the output is modeled *explicitly* and various ways to separate it from the system disturbance-free dynamics. Available for identification are the excitation signals and the resultant responses, which may be corrupted by unknown and possibly dominating disturbances. We assume that no actual measurement of the disturbances themselves is available for identification. The case where the disturbance profile is unknown and may be quite complicated, but its period is known, is considered first. Next, we consider the case where the disturbance frequencies are known only approximately, and a procedure is applied to iteratively refine the estimation of the disturbance frequencies for successful system identification. Finally, we examine the situation where the unknown disturbance is nonperiodic. The identification produces results that can be used for the synthesis of feedforward and feedback control to cancel the disturbance effect. Both simulation and experimental results will be used for illustration. The simulation is carried out with a model of a communications satellite, and the experimental results are obtained from a flexible truss structure. A companion paper addresses the system identification problem where the disturbance effect is modeled implicitly.

Introduction

THE control problem of rejecting unwanted disturbance has many applications in electrical, mechanical, aerospace, and acoustic systems. Many methods have been developed to treat this problem. If both the plant and disturbance frequencies are known, classical feedback control approaches call for the design of filters with high gain at the disturbance frequencies to produce corresponding zeros in the closed-loop transfer function relating the disturbances to the system response. The closed-loop system is then capable of rejecting the unwanted disturbances at the designed frequencies. One such design is included in the attitude control systems of the INMARSAT III and INTELSAT VIII spacecraft.[1] Rejection of sinusoidal disturbances can also be achieved using the frequency-shaped cost functional method.[2] This is an adaptation of linear quadratic Gaussian (LQG) design methods that determines the control necessary to minimize a cost function expressed in the frequency domain. The cost function penalizes the response at the known disturbance frequencies, resulting in high gain feedback at these frequencies. Another method for disturbance rejection is known as disturbance accommodation control or, as it is sometimes called, disturbance modeling or disturbance estimation.[3] It assumes an effective disturbance input at the control input location that has the same effect at the output as the actual disturbance. The same harmonic disturbance can be modeled as marginally stable modes and appended to the state-space model of the plant. An observer is designed to estimate the plant states together with the disturbance from which the control signal is computed. The disturbance observer approach also assumes an equivalent effective disturbance at the control input, but it uses an inverse plant model to estimate this disturbance for control.[4] Repetitive control is another approach to reject periodic disturbance where the tracking error observed in the previous periods is used to correct the control signal for the current period.[5,6] Adaptive control can be used to cancel the effects of unknown sinusoidal or periodic disturbance through the use of a sufficiently overparameterized model so that the disturbance dynamics can be entirely absorbed in the identified model.[7] With the disturbance embedded in the model, based on the internal model principle the resulting feedback control has infinite open-loop gain at the disturbance frequencies and achieves complete cancellation. The filtered-X least mean squares is another well-known disturbance rejection method.[8] This method requires measurement of a disturbance-correlated signal, and together with an assumed model of the system it adaptively tunes the coefficients of a finite impulse response filter driven by a disturbance-correlated reference signal to achieve disturbance cancellation. Adaptive inverse control combines adaptive feedforward techniques for command or model following and adaptive feedback techniques for disturbance rejection.[9] Artificial neural networks can also be used to provide disturbance rejection control.[10] Neural networks are used to identify the dynamics from the disturbance sources, and the control inputs to the system outputs and adapts the control signals for disturbance cancellation.

Instead of treating the disturbance-rejection problem from a control system synthesis standpoint, we address the problem from a system identification perspective. The research first focuses on the system identification problem in the presence of unknown disturbance inputs, and then uses the identification results to solve the related disturbance-rejection control problem. We assume no a priori knowledge of the system and no actual measurement of the disturbance input. Rather than calculating the actual disturbances, we identify their effect on the system output. As a result, the number of disturbances or where they enter the system is unimportant. Unlike adaptive filtering or neural network approaches, we do not require a disturbance-correlated reference signal or need to determine the transfer function relating the disturbances to the system response. We assume that the only data available are the excitation at the control excitation inputs and the disturbance-corrupted response.

Specifically, we consider the situation where the disturbance effect on the system output is modeled explicitly and examine various ways to separate it from the system disturbance-free dynamics without actually knowing the disturbance input itself. The situation where the disturbance effect is modeled implicitly is investigated in Ref. 11. The following cases are considered in this paper. First, the actual profile of a periodic disturbance is unknown and may be quite complicated, but its period is known. This information can often be known or experimentally determined, especially in high-precision computer-controlled processes. For example, a spacecraft may be subject to periodic disturbance arising from repetitive thruster firing for orbit control or from scanning payloads. Here the thrusters are commanded to fire precisely at regular predetermined intervals, and in the case of scanning payloads the period of the scanning motion

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is known and maintained very precisely by digital signal processors. Although the actual disturbance profile that the scanner imparts on the spacecraft is not known, its scanning period is known. As another example, in the timing belt drive system of high-precision color imaging equipment, an index pulse is used to signal the duration of each repetitive cycle from which the period of the disturbance source is determined.[12] If the disturbance period is known or can be measured, we will show that it is possible to identify the system and the disturbance effect directly without any need to resolve the disturbance profile into its constitutive harmonic components. This is particularly convenient if the disturbance profile is complicated, and hence its spectral content is rich. Second, we consider the case where the disturbance period and its harmonic components are only known approximately. A simple procedure is used to iteratively estimate the individual disturbance frequencies for successful system identification. Third, we examine the extreme case where the unknown disturbance is nonperiodic. This makes the identification problem mathematically ill-posed, and we will study how to handle such a situation. Both simulation and experimental results will be provided to illustrate the developed procedures.

Mathematical Formulation

We start by showing the basic difficulties associated with the separation of the system dynamics from the effect of the unknown disturbances. The limitations that are apparent from this preliminary analysis motivate the subsequent development of a solution to the identification problem.

System and Disturbance Identification from Steady-State Data

The system to be identified and controlled is assumed to be representable by a linear discrete-time state-space model

$$x(k+1) = Ax(k) + Bu(k) + B_d d(k), \qquad y(k) = Cx(k) \quad (1)$$

where $x(k)$ is an $n \times 1$ state vector, $u(k)$ is the $m \times 1$ control and excitation vector, $y(k)$ is the $q \times 1$ output vector, and k is the time step. The vector $d(k)$ represents the unknown disturbance inputs. The matrices A, B, C, B_d, and the state vector's exact dimension n are unknown. Only measurements of the input $u(k)$ and system response $y(k)$ are available for identification and control. Starting with Eq. (1), the system response at any time step k can be written as

$$y(k) = CA^k x(0) + CBu(k-1) + CABu(k-2) + \cdots + CA^{k-1}Bu(0) + y_d(k) \quad (2)$$

where $y_d(k)$ defines the contribution of the disturbance on the system output

$$y_d(k) = CB_d d(k-1) + CAB_d d(k-2) + \cdots + CA^{k-1}B_d d(0) \quad (3)$$

The overall response consists of the effect of the initial condition, the excitation input, and the disturbance input. At issue is whether or not these individual contributions can be separated using data consisting of the excitation input and disturbance-corrupted data only. From the point of view of state-space system identification, the goal is to identify the system Markov parameters $(CB, CAB, \ldots, CA^{k-1}B, \ldots)$. The Markov parameters completely describe the system (disturbance-free) input-output dynamics because they are its unit pulse response samples from which a state-space representation of the system can be constructed. From Eq. (2) it is not immediately obvious that the identification can be carried out in general because both the initial condition and the disturbance input are unknown. However, for the moment, we consider asymptotically stable systems so that in the steady-state the initial condition response can be neglected, the disturbance response becomes periodic with a common period N, and the sequence of system Markov parameters can be truncated after a finite number of terms, say p_s. Thus, in the steady state

$$y(k) \approx CBu(k-1) + CABu(k-2) + \cdots + CA^{p_s-1}Bu(k-p_s) + y_d(k), \qquad k \geq p_s \quad (4)$$

$$y_d(k) \approx CB_d d(k-1) + CAB_d d(k-2) + \cdots + CA^{p_s-1}B_d d(k-p_s), \qquad k \geq p_s \quad (5)$$

If the common period N is known, then with a sufficient amount of steady-state data, the system Markov parameters and the disturbance response can be easily separated, provided that the excitation is sufficiently rich so that V is full row rank:

$$Y = yV^T(VV^T)^{-1} \quad (6)$$

where

$$Y = \begin{bmatrix} CB & CAB & \cdots & CA^{p_s-1}B & y_d(p_s) & y_d(p_s+1) & \cdots & y_d(p_s+N-1) \end{bmatrix} \quad (7)$$

$$y = [y(p_s) \quad y(p_s+1) \quad \cdots \quad y(p_s+\ell-1)] \quad (8)$$

$$V = \begin{bmatrix} u(p_s-1) & u(p_s) & \cdots & u(p_s+N-2) & u(p_s+N-1) & u(p_s+N) & \cdots & u(p_s+\ell-2) \\ u(p_s-2) & u(p_s-1) & \cdots & u(p_s+N-3) & u(p_s+N-2) & u(p_s+N-1) & \cdots & u(p_s+\ell-3) \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ u(0) & u(1) & \cdots & u(N-1) & u(N) & u(N+1) & \cdots & u(\ell-1) \\ 1 & 0 & \cdots & 0 & 1 & 0 & \cdots & \cdots \\ 0 & 1 & \cdots & 0 & 0 & 1 & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \\ 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & \cdots \end{bmatrix} \quad (9)$$

Although straightforward, the described procedure has several major limitations. For lightly damped systems (e.g., spacecraft with flexible appendages), the number of Markov parameters needed to model the system is very large. Even after reaching steady state, a large amount of additional data would be needed to identify the Markov parameters and disturbance response. These limitations can be overcome if a mechanism can be found so that such reliance on steady-state data and on the system actual damping can be eliminated. Indeed, this is possible as shown in the following development.

System and Disturbance Identification via an Implicit Observer

For lightly damped systems the Markov parameter model of Eq. (4) is cumbersome because it calculates a one-step-ahead response prediction using a potentially unlimited number of past input values. A more efficient approach is to base the response prediction on both past input and past output values. This can be done by manipulating Eq. (1) as follows:

$$x(k+1) = Ax(k) + Bu(k) + B_d d(k) + My(k) - My(k)$$
$$= (A + MC)x(k) + Bu(k) - My(k) + B_d d(k) \tag{10}$$

Defining

$$\bar{A} = A + MC, \qquad \bar{B} = [B, -M], \qquad v(k) = \begin{bmatrix} u(k) \\ y(k) \end{bmatrix}$$

we have a counterpart to Eq. (1):

$$x(k+1) = \bar{A}x(k) + \bar{B}v(k) + B_d d(k), \qquad y(k) = Cx(k) \tag{11}$$

which produces the counterparts to Eqs. (2) and (3):

$$y(k) = C\bar{A}^k x(0) + CBv(k-1) + C\bar{A}Bv(k-2) + \cdots + C\bar{A}^{k-1}Bv(0) + \eta(k) \tag{12}$$

$$\eta(k) = CB_d d(k-1) + C\bar{A}B_d d(k-2) + \cdots + C\bar{A}^{k-1}B_d d(0) \tag{13}$$

Up to this point, the matrix M is arbitrary. Now it will be used to our advantage by making $\bar{A}$ into a nilpotent matrix,

$$\bar{A}^p = (A + MC)^p \equiv 0 \tag{14}$$

so that Eqs. (12) and (13) become

$$y(k) = C\bar{B}v(k-1) + C\bar{A}\bar{B}v(k-2) + \cdots + C\bar{A}^{p-1}\bar{B}v(k-p) + \eta(k), \qquad k \ge p \tag{15}$$

$$\eta(k) = CB_d d(k-1) + C\bar{A}B_d d(k-2) + \cdots + C\bar{A}^{p-1}B_d d(k-p), \qquad k \ge p \tag{16}$$

In Eq. (15) the term $\eta(k)$ is referred to as the *disturbance effect*. Let us now examine the significance of the preceding operation. First, imposing the condition of Eq. (14) eliminates the explicit dependence on the unknown initial condition $x(0)$. Second, it compresses a potentially unlimited number of system Markov parameters CB, $CAB, \ldots, CA^{p_s-1}B$ to a finite set $C\bar{B}, C\bar{A}\bar{B}, \ldots, C\bar{A}^{p-1}\bar{B}$ where $p \ll p_s$. In fact, p now depends on the true order of the system and the number of outputs rather than the stability of the system (the selection of p will be discussed later). Third, imposing Eq. (14) turns the disturbance response $y_d(k)$, which becomes periodic only in the steady state, into $\eta(k)$, which becomes periodic in exactly p time steps regardless of the original system dynamics. This is because $\eta(k)$ is the actual disturbance $d(k)$ driven through a dynamic system specified by $\bar{A}$, B_d, C, with the special property that $\bar{A}^k = 0$ for $k \ge p$, i.e.,

$$x(k+1) = \bar{A}x(k) + B_d d(k), \qquad \eta(k) = Cx(k) \tag{17}$$

Fourth, non-steady-state data can now be used for system identification, and because Eq. (14) is an exact relation, the model given by Eq. (15) is exact. Its earlier counterpart, Eq. (4), is only approximate because of the theoretically nonzero truncation error introduced when the contribution of the initial condition in the steady state is neglected.

At this point several questions arise with regard to the existence of M and p. Note that the combination $A + MC$ has the exact form of the system matrix of an observer where M is an observer gain. From discrete-time observer pole placement theory it is known that such a matrix M satisfying Eq. (14) places the eigenvalues of $A + MC$ at the origin in the complex plane and is guaranteed to exist as long as the pair A, C is observable. This condition is automatically satisfied because in system identification only the observable part of the system can be identified from input-output data. Also, the condition on p is such that it at least exceeds the minimum value $p_{\min}$ defined to be the smallest integer value such that $qp_{\min} \ge n$ (Ref. 13). Although $p_{\min}$ defines the minimum value for p, there is no need to select this minimum value. The advantages of M can be seen by a simple example. For a single-input single-output system with a dominant 1 rad/s mode, 0.1% damping, and a 2-Hz sample rate, 10,000 Markov parameters would be required for Eq. (4) to be valid with $p_s = 10{,}000$. Now for the model of Eq. (15), $p_{\min} = 2$, and in theory any $p \ge p_{\min}$ would suffice. The advantage of using p larger than $p_{\min}$ in the presence of noise will be discussed in a later section.

The next important question is how the condition in Eq. (14) can be imposed when A and C are not known. Having justified the existence of M, the problem is not finding M given A and C as in the standard observer design problem, but rather it is finding A, C while simultaneously imposing Eq. (14). This can be done by using input-output data to solve for the combinations $C\bar{B}, C\bar{A}\bar{B}, \ldots, C\bar{A}^{p-1}\bar{B}$ and $\eta(k)$ subject to the usual conditions that the excitation is sufficiently rich, and the data record is sufficiently long,

$$\bar{Y} = y\bar{V}^T(\bar{V}\bar{V}^T)^+ \tag{18}$$

where + denotes the pseudoinverse computed via the singular value decomposition and

$$\bar{Y} = [C\bar{B} \quad C\bar{A}\bar{B} \quad \cdots \quad C\bar{A}^{p-1}\bar{B} \quad \eta(p) \quad \eta(p+1) \quad \cdots \quad \eta(p+N-1)] \tag{19}$$

$$y = [y(p) \quad y(p+1) \quad \cdots \quad y(p+\ell-1)] \tag{20}$$

$$\bar{V} = \begin{bmatrix} v(p-1) & v(p) & \cdots & v(p+N-2) & v(p+N-1) & v(p+N) & \cdots & v(p+\ell-2) \\ v(p-2) & v(p-1) & \cdots & v(p+N-3) & v(p+N-2) & v(p+N-1) & \cdots & v(p+\ell-3) \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ v(0) & v(1) & \cdots & v(N-1) & v(N) & v(N+1) & \cdots & v(\ell-1) \\ 1 & 0 & \cdots & 0 & 1 & 0 & \cdots & \cdots \\ 0 & 1 & \cdots & 0 & 0 & 1 & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \\ 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & \cdots \end{bmatrix} \tag{21}$$

Finally, because the goal of system identification is to recover the actual system Markov parameters $CB, CAB, CA^2B, \ldots,$ not the parameter combinations $C\bar{B}, C\bar{A}\bar{B}, \ldots, C\bar{A}^{p-1}\bar{B}$, one must examine whether the former sequence can be recovered uniquely from the latter. This can indeed be done as follows. Define $\beta_k = C\bar{A}^{k-1}B$, $\alpha_k = -C\bar{A}^{k-1}M$ so that $\bar{Y}(k) = C\bar{A}^{k-1}[B \; -M] = [\beta_k \; \alpha_k]$, $k = 1, 2, \ldots, p$. By algebraic manipulation a recursive expression can be derived to calculate an arbitrary number of system Markov parameters $Y(k) = CA^{k-1}B$ starting with $Y(1) = \beta_1$, and

$$Y(k) = \beta_k + \sum_{i=1}^{k-1} \alpha_i Y(k-i), \qquad k = 2, 3, \ldots, p \quad (22)$$

$$Y(k) = \sum_{i=1}^{p} \alpha_i Y(k-i), \qquad k = p+1, p+2, \ldots \quad (23)$$

As already mentioned, the system Markov parameters completely define the dynamics from the control input to the system output. The system Markov parameters can be factored to produce a minimum-order state-space representation for A, B, C. The analytical solution to this step is exact and straightforward, and the formulas for several such realizations are given in Ref. 14.

Relationship to a Kalman Filter in the Presence of Noise

In this section we provide further analysis of the preceding operation by relating it to an optimal Kalman filter, yielding additional insights on the role of M and p in the presence of noise. Consider the case where the original system is contaminated by process and measurement noise,

$$x(k+1) = Ax(k) + Bu(k) + B_d d(k) + w_1(k)$$
$$y(k) = Cx(k) + w_2(k) \quad (24)$$

where $w_1(k)$ also incorporates any random variations in $u(k)$ and $d(k)$. As usual, the standard assumptions about $w_1(k)$ and $w_2(k)$ are made: they are stationary, zero-mean, Gaussian, white, and independent. If A is stable and A, C is an observable pair, then the preceding system admits an optimal steady-state Kalman filter of the form[15]

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + B_d d(k) - K\varepsilon(k)$$
$$y(k) = C\hat{x}(k) + \varepsilon(k) \quad (25)$$

The steady-state Kalman filter gain K depends on the system parameters and statistics of the random noise processes. Because $d(k)$ is not known, it is not possible to implement the preceding Kalman filter, but for our present purpose we are only interested in the existence and the structure of the resultant equations. Equation (25) can be manipulated to yield

$$\hat{x}(k+1) = (A + KC)\hat{x}(k) + Bu(k) - Ky(k) + B_d d(k)$$
$$y(k) = C\hat{x}(k) + \varepsilon(k) \quad (26)$$

Define $\tilde{A} = A + KC$, $\tilde{B} = [B, -K]$, and $\tilde{p}$ sufficiently large such that $(A + KC)^{\tilde{p}} \approx 0$, then for $k \geq \tilde{p}$ the input-output form of Eq. (26) takes the form

$$y(k) \approx C\tilde{B}v(k-1) + C\tilde{A}\tilde{B}v(k-2) + \cdots + C\tilde{A}^{\tilde{p}-1}\tilde{B}v(k-\tilde{p}) + \tilde{\eta}(k) + \varepsilon(k), \qquad k \geq \tilde{p} \quad (27)$$

$$\tilde{\eta}(k) \approx CB_d d(k-1) + C\tilde{A}B_d d(k-2) + \cdots + C\tilde{A}^{\tilde{p}-1}B_d d(k-\tilde{p}), \qquad k \geq \tilde{p} \quad (28)$$

Now, observe that Eqs. (27) and (28) have the exact form and internal structure as Eqs. (15) and (16), except K and $\tilde{p}$ now play the role of M and p. Thus in the presence of noise, if p is sufficiently large such that the transient response of the associated Kalman filter is negligible, the resultant input-output map as given in Eq. (27) will only be driven by the Kalman filter residual, which is known to be white, zero-mean, and uncorrelated with the measurements. This fact is significant because the least-squares solution as given in Eq. (18) for such a white-noise contaminated input-output map is known to be unbiased and consistent; i.e., the covariance of the estimation error converges to zero as the data length tends to infinity. In practice, this implies that more accurate identification can be expected as p is increased. This result will be illustrated using experimental data. In the absence of noise, there is no Kalman filter, but one instead has an observer. The deterministic (noise-free) analysis can be carried out similarly revealing that the matrix M is a deadbeat observer gain. Equation (10) becomes an observer equation when $x(k)$ is replaced by $\hat{x}(k)$ and Eq. (14) is satisfied exactly.

Estimation of System Response to Unknown Disturbances

Note that Eq. (15) is an extension of the standard auto-regressive model with eXogenous input model to include the disturbance effect

$$y(k) = \alpha_1 y(k-1) + \cdots + \alpha_p y(k-p) + \beta_1 u(k-1) + \cdots + \beta_p u(k-p) + \eta(k) \quad (29)$$

In this paper we have established the connection between the coefficients of this input-output model and those of the original state-space model in terms of an observer (or Kalman filter) gain, and the precise nature of the disturbance effect term, which is periodic in exactly p time steps regardless of the system stability. This is in contrast to the usual connection between the two types of models through observable or controllable canonical representations. Once the coefficients of this model and the disturbance effect terms are identified from disturbance-corrupted data as in Eq. (18), the contribution to the system response by the unknown periodic disturbances, defined as $y_d(k)$ in Eq. (5), can be calculated from

$$y_d(k) = \alpha_1 y_d(k-1) + \alpha_2 y_d(k-2) + \cdots + \alpha_p y_d(k-p) + \eta(k) \quad (30)$$

An estimate of the disturbance response can be obtained by propagating Eq. (30) starting from $k = p$ with $y_d(p-1) = y_d(p-2) = \cdots = y_d(0) = 0$, using $\eta(k)$ determined from Eq. (18). The estimated disturbance response will match the actual disturbance response $y_d(k)$ in the steady state. Alternatively, making use of the fact that $y_d(k) = y_d(k+N)$ in the steady state, the N samples of the steady-state response of the system caused by the unknown disturbances can be solved for directly.

Disturbance Effect Modeling by Basis Functions

The last N rows of $\bar{V}$ in Eq. (21) can be viewed as orthogonal basis vectors that can model any arbitrary function of period N, whose profile may be very complex. If the period of the disturbance input is known, then this particular formulation is particularly advantageous because it bypasses the need to resolve the disturbance into its constitutive harmonic components whose number may be inconveniently large. This would be the case if the disturbances have many sharp jumps or discontinuities.

If, however, the disturbances contain only a small number of frequencies, then the dimension of this basis is unnecessarily large. A more efficient approach is to model one period of $\eta(k)$ by a number of basis vectors, say L, where $L \ll N$. This reduces the number of unknowns and the data required. Basis vectors can be generated using any sequence of orthogonal functions, e.g., sines and cosines, orthogonal polynomials, wavelets, etc., with the best choice depending on the characteristics of the disturbance waveform. Let $\phi_i(k)$ denote the basis vectors with the associated coefficients γ_i, $i = 1, 2, \ldots, L$. In terms of these basis vectors, $\eta(k)$ can be approximated by $\eta(k) \approx \gamma_1\phi_1(k-p) + \cdots + \gamma_L\phi_L(k-p)$, $k \geq p$. Using this expression, $\bar{Y}$ can now be determined from Eq. (18), where

$$\bar{Y} = \begin{bmatrix} C\bar{B} & C\bar{A}\bar{B} & \cdots & C\bar{A}^{p-1}\bar{B} & \gamma_1 & \gamma_2 & \cdots & \gamma_L \end{bmatrix} \quad (31)$$

$$\bar{V} = \begin{bmatrix} v(p-1) & v(p) & \cdots & v(p+N-2) & v(p+N-1) & v(p+N) & \cdots & v(p+\ell-2) \\ v(p-2) & v(p-1) & \cdots & v(p+N-3) & v(p+N-2) & v(p+N-1) & \cdots & v(p+\ell-3) \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ v(0) & v(1) & \cdots & v(N-1) & v(N) & v(N+1) & \cdots & v(\ell-1) \\ \phi_1(0) & \phi_1(1) & \cdots & \phi_1(N-1) & \phi_1(0) & \phi_1(1) & \cdots & \cdots \\ \phi_2(0) & \phi_2(1) & \cdots & \phi_2(N-1) & \phi_2(0) & \phi_2(1) & \cdots & \cdots \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \cdots \\ \phi_L(0) & \phi_L(1) & \cdots & \phi_L(N-1) & \phi_L(0) & \phi_L(1) & \cdots & \cdots \end{bmatrix} \tag{32}$$

Identification with Uncertain Disturbance Frequencies

Another case of interest occurs when the constitutive disturbance frequencies (hence its common period) are known only approximately. By using the basis function concepts, it is possible to determine these disturbance frequencies iteratively. In the following we describe a simple procedure for a single disturbance frequency. Generalization to multiple disturbance frequencies will be obvious. For a periodic disturbance appropriate basis functions are the sine and cosine functions. Let ω_1 and ω_2 denote lower and upper bounds on the actual disturbance frequency, i.e., $\omega_1 \leq \omega \leq \omega_2$. We select S distinct frequencies during the prescribed intervals from which $2S$ basis functions (one sine and one cosine for each frequency) associated with these frequencies are generated. The identification of the corresponding coefficients describing the disturbance effect can be carried out where $\bar{V}$ now assumes the form

$$\bar{V} = \begin{bmatrix} v(p-1) & v(p) & \cdots & \cdots & \cdots & v(p+\ell-2) \\ v(p-2) & v(p-1) & \cdots & \cdots & \cdots & v(p+\ell-3) \\ \vdots & \vdots & & \vdots & & \vdots \\ v(0) & v(1) & \cdots & \cdots & \cdots & v(\ell-1) \\ \psi_1(0) & \psi_1(1) & \cdots & \cdots & \cdots & \psi_1(\ell-1) \\ \psi_2(0) & \psi_2(1) & \cdots & \cdots & \cdots & \psi_2(\ell-1) \\ \vdots & \vdots & \cdots & \vdots & \cdots & \cdots \\ \psi_S(0) & \psi_S(1) & \cdots & \cdots & \cdots & \psi_S(\ell-1) \end{bmatrix} \tag{33}$$

where $\psi_i(k) = [\sin(\omega_i k\Delta t), \cos(\omega_i k\Delta t)]^T$, $i = 1, 2, \ldots, S$, and Δt is the sampling interval. Because the system dynamics is primarily absorbed by the coefficients α_i and β_i, $i = 1, 2, \ldots, p$, and the disturbance effect by the disturbance model, the coefficients of the basis vectors whose frequencies are closer to the true disturbance frequencies, will be larger. This information can be used to improve the estimates further until the desired accuracy is achieved. Typically, only a few iterations are needed. This procedure will be illustrated on actual experimental data later in the paper.

Identification in the Presence of Nonperiodic Disturbances

Another interesting application of the basis function approach is that it allows the removal of the effect of certain nonperiodic and unknown disturbances corrupting the measured system response. Identification in the presence of unknown and nonperiodic disturbances (except the case of random noise) is an ill-posed problem. Therefore, it is not expected in general that the system can still be identified correctly in this case. However, if the nonperiodic disturbance effect can be modeled by a set of basis functions, then it is possible to separate the system dynamics from that of the disturbance. Similarly, if the output is corrupted by a time-varying sensor bias, this bias can be removed as well. Suitable basis functions that can be used to model a nonperiodic signal include the Legendre's polynomials, wavelets, or others. The wavelets tend to be a better choice because they have compact support and they do not suffer from endpoint conditions as do the Legendre's polynomials. The identification using any of these nonperiodic basis functions can be carried out similarly as before. The matrix $\bar{V}$ assumes the same form as that given in Eq. (33), except the sine and cosine basis functions are now replaced by these nonperiodic basis functions. An example will be presented to illustrate the applicability of this approach to remove a dominant time-varying sensor bias on actual experimental data.

Feedforward Control for Periodic Disturbance Cancellation

The system identification emphasis helps simplify the control problem. No measurements of the actual disturbances $d(k)$ or the transfer functions relating the disturbances to the system response are needed. From Eq. (29) the feedforward control $u_f(k)$ that cancels the system response caused by the disturbances in the steady state must satisfy

$$\beta_1 u_f(k-1) + \beta_2 u_f(k-2) + \cdots + \beta_p u_f(k-p) + \eta(k) = 0 \tag{34}$$

There are three ways to solve for the feedforward control signal that cancel the disturbance effect on the system output. Here we outline the key elements of each approach.

First, the simplest approach is to propagate Eq. (34) recursively forward in time, and in the steady state the needed feedforward control signal is obtained. This approach, however, is not suitable for non-minimum-phase systems (or identification models) because such a recursion will cause $u_f(k)$ to grow unbounded. Second, this problem can be bypassed by using the knowledge that the control signal needed to cancel a periodic disturbance is also a periodic signal of the same period $u_f(k) = u_f(k+N)$, where N is either known or determined experimentally. Using this periodicity condition together with Eq. (34), the N discrete values of the feedforward control signal can be solved for directly.[16] Third, when there are only a small number of disturbance frequencies, the form of the feedforward control can be imposed a priori,

$$u_f(k) = \sum_{i=1}^{n_f} G_i \sin(\omega_i k\Delta t) + H_i \cos(\omega_i k\Delta t) \tag{35}$$

where n_f is the number of disturbance frequencies. Substituting Eq. (35) into Eq. (34), the coefficients defining the feedforward control signal can be easily computed. The first approach is suitable for minimum-phase systems or identification models. The second approach is suitable if the common disturbance period is known or can be determined. This approach bypasses the need to resolve the disturbance frequencies, and its computational cost is independent of the complexities of the periodic disturbance profile. The third approach is suitable if one wishes to reject a specific number of disturbance frequencies. Both the second and third approaches are suitable for non-minimum-phase systems because the imposed form of the solution prevents the control from growing unbounded, and either by a batch type or recursive solution technique can be used. Whether a feedforward control signal exists that can exactly cancel the disturbance depends on several factors. If the disturbance causes the output to have a frequency component at which the control input has no influence (e.g., the input-output transfer function equals zero at this frequency), then this component cannot be canceled by feedforward control. In addition, provided the control can influence the system output, if the number of independent inputs is equal to or greater than the number of outputs, then a feedforward control exists that exactly cancels the disturbance response. Otherwise, a perfect cancellation is in general not possible.

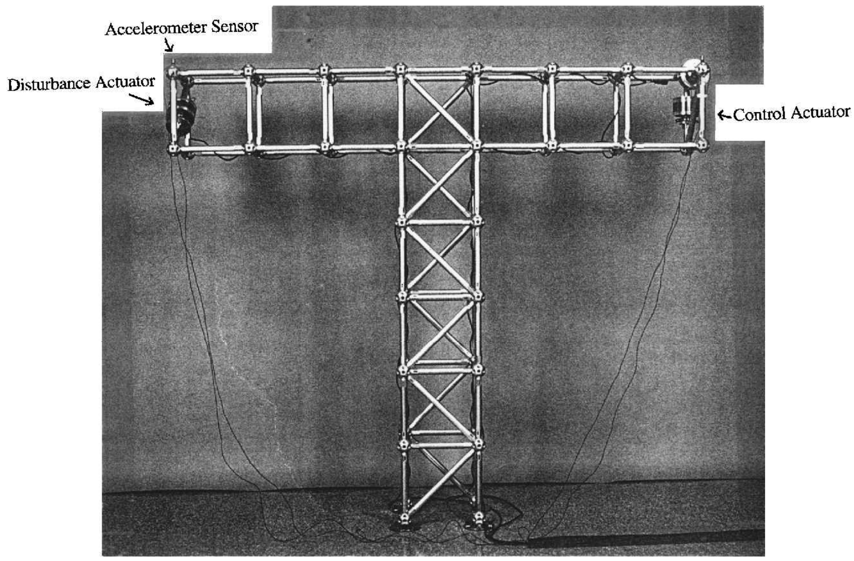

Fig. 1 Flexible truss structure.

Combined Feedforward and Feedback Control

To provide additional attenuation of the disturbance effect when feedforward control is imperfect, feedback control can be used. Note that the identification also yields information needed to design a feedback controller. One approach is to develop a state-space model from the system Markov parameters given by Eqs. (22) and (23) and then design a suitable state-space controller. Another approach is to design a controller from the identified input-output model directly. Although the system dynamics is modified by the feedback control, the feedforward control signal needed to cancel the steady-state effect of the disturbance remains unchanged. The two types of control signals can be determined independently and applied simultaneously without affecting one another. This can be shown as follows. Let $u(k)$ denote the sum of the feedforward and feedback control signals $u(k) = u_f(k) + u_b(k)$, then Eq. (29) becomes

$$y(k) = \alpha_1 y(k-1) + \cdots + \alpha_p y(k-p) + \beta_1 u_b(k-1) + \cdots + \beta_p u_b(k-p) + \beta_1 u_f(k-1) + \cdots + \beta_p u_f(k-p) + \eta(k) \tag{36}$$

With the feedforward control determined from Eq. (34), Eq. (36) becomes

$$y(k) = \alpha_1 y(k-1) + \cdots + \alpha_p y(k-p) + \beta_1 u_b(k-1) + \cdots + \beta_p u_b(k-p) \tag{37}$$

Therefore, with the feedforward control satisfying Eq. (34), the system behaves as if the disturbances were not present regardless of the method used to compute the feedback control $u_b(k)$.

Illustrative Examples

Both experimental and simulation results are provided to illustrate the system identification, disturbance modeling, and disturbance rejection approach as just presented. The experimental results are obtained from a flexible truss structure at Princeton University (Fig. 1). The acceleration response, measured at one end of the structure, is filtered to remove sensor drift resulting in a velocity representative signal for identification and control. The disturbance is generated by a proof-mass actuator in close proximity to the acceleration sensor. The control is applied by another proof-mass actuator located at the other end of the structure. This configuration allows direct coupling of the disturbance to the sensor while the control actuator is located far away from the sensor, making the identification and control problem more difficult. The simulation result is presented to illustrate how the method may be applicable to the identification and control of a modern communications satellite.

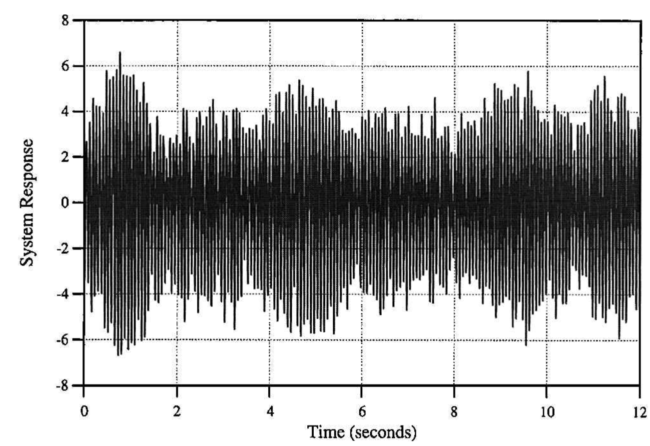

Fig. 2 Disturbance-corrupted response.

System Identification in the Presence of Periodic Disturbances

For identification purposes a random excitation signal is injected through the control actuator while the disturbance is acting. Figure 2 shows the resulting disturbance-corrupted system response. With this excitation and response data we wish to isolate the true excitation-to-output dynamics from that induced by the disturbance. The disturbance consists of two frequency components producing a dominant effect on the system response. The identification was performed using 2128 input-output data points (sampling period of 0.006 s). A basis of 50 orthogonal sines and cosines from 12 to 16 Hz bracketing the first disturbance frequency with a spacing of 0.0814 Hz is used with $p = 80$. Examining the norms of the basis vector coefficients, the first disturbance frequency is determined to lie between 13.87 and 13.95 Hz because these frequencies have the largest coefficients. To refine the frequency estimate, a second iteration is performed using vectors at these two bracketing frequencies with one inserted in between to produce an estimate of 13.953 Hz corresponding to the largest coefficient in this iteration. This value is within 0.015% of the true frequency of 13.951 Hz (this value is chosen intentionally so that no basis vectors coincide with the disturbance frequency). The second disturbance frequency is detected simultaneously with the first and is found to be 27.914 Hz, within 0.05% of the true frequency of 27.902 Hz. Further iterations could be used for better frequency estimates, but it is not necessary because this accuracy is adequate for successful identification and control.

To verify that the identification model obtained with disturbance-corrupted data is valid, its pulse response is compared against that of a reference model identified from data with the disturbance source

Table 1 Identification results with various values of p

	p = 30		p = 50		p = 80		Reference values[a]	
System mode	Frequency, Hz	Damping ratio	Frequency, Hz	Damping ratio	Frequency, Hz	Damping ratio	Frequency, Hz	Damping ratio
1	7.52	0.0898	7.51	0.0593	7.34	0.0291	7.30	0.0278
2	13.59	0.0121	13.49	0.0102	13.51	0.0083	13.50	0.0086
3	17.54	0.0110	17.57	0.0091	17.54	0.0077	17.59	0.0077
4	43.57	0.0110	43.76	0.0089	43.78	0.0081	43.91	0.0076
5	48.65	0.0129	48.64	0.0093	48.68	0.0080	48.80	0.0086
6	64.92	0.0143	65.05	0.0144	64.96	0.0135	64.71	0.0123

[a]The reference system dynamics is determined from data collected without the disturbance acting.

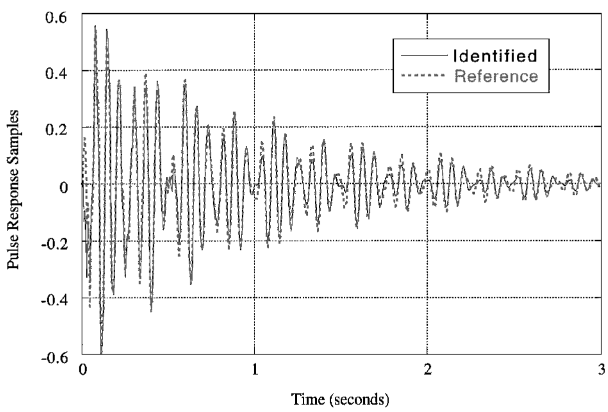

Fig. 3 Pulse responses identified with and without disturbance acting.

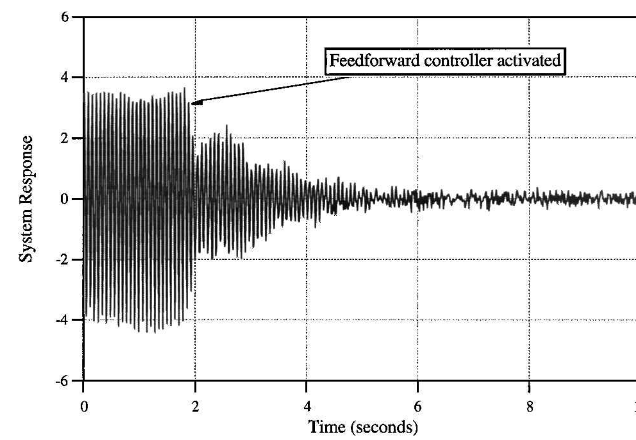

Fig. 5 Response of structure to feedforward control.

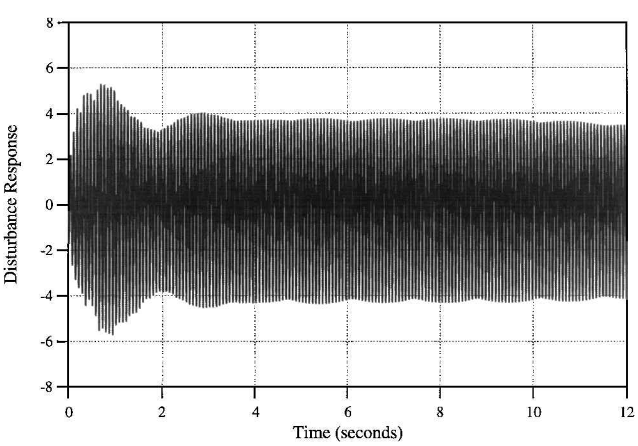

Fig. 4 Identified contribution of disturbance on system response.

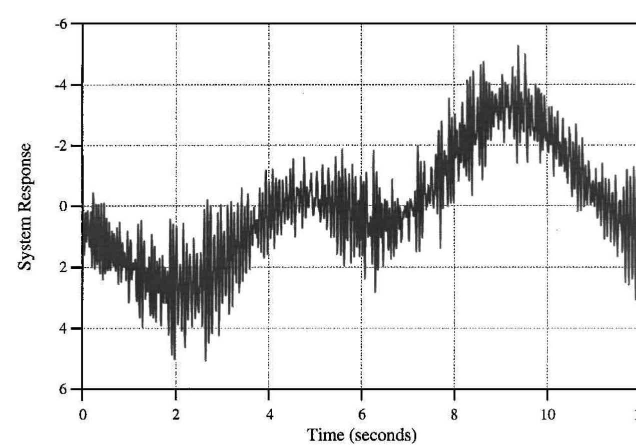

Fig. 6 Measurement corrupted by time-varying nonperiodic bias.

turned off (Fig. 3). The identification results also allow us to compute the effect of the disturbance on the overall system response, which is referred to as $y_d(k)$ in the mathematical development (Fig. 4).

To justify the choice of $p = 80$, which is quite adequate for this application, Table 1 summarizes the identification results obtained with disturbance-corrupted data for $p = 30$, 50, and 80 vs those from a reference model identified with disturbance-free data for $p = 80$. Consistent with the developed theory, as p increases the disturbance-corrupted identification results converge to those obtained with disturbance-free data. The effect is particularly remarkable for the damping ratio estimates, which are commonly known be very sensitive to noisy experimental data. Also, note that although the identification results improve with increasing p a wide range of p can be selected without significantly altering the results.

Feedforward Disturbance Rejection

The validity of the identification result is further verified by using it to synthesize a feedforward control signal to cancel the disturbance effect. Figure 5 shows the system response to feedforward control calculated by solving Eq. (35) recursively in real time using the disturbance frequency estimates just given and the identified model coefficients. Once control is applied, the response quickly decays to near zero. The source of the residual is primarily caused by the random motion transmitted to the truss structure from its base attachment.

System Identification in the Presence of a Time-Varying Measurement Bias

System identification in the presence of a simulated slowly time-varying nonperiodic measurement bias for the truss structure is presented here. Figure 6 shows the biased response to a random excitation input similar to that already used. To separate the bias from the system input-output dynamics, a basis of 64 orthogonal wavelets is constructed on an interval of 2048 data points (0.006-s sample period). These wavelets absorb the effect of the measurement bias so that the system input-output dynamics is correctly identified. This is confirmed by the close agreement between the identified frequency response function and that of the reference model just described, indicating the identification is successful and the time-varying bias

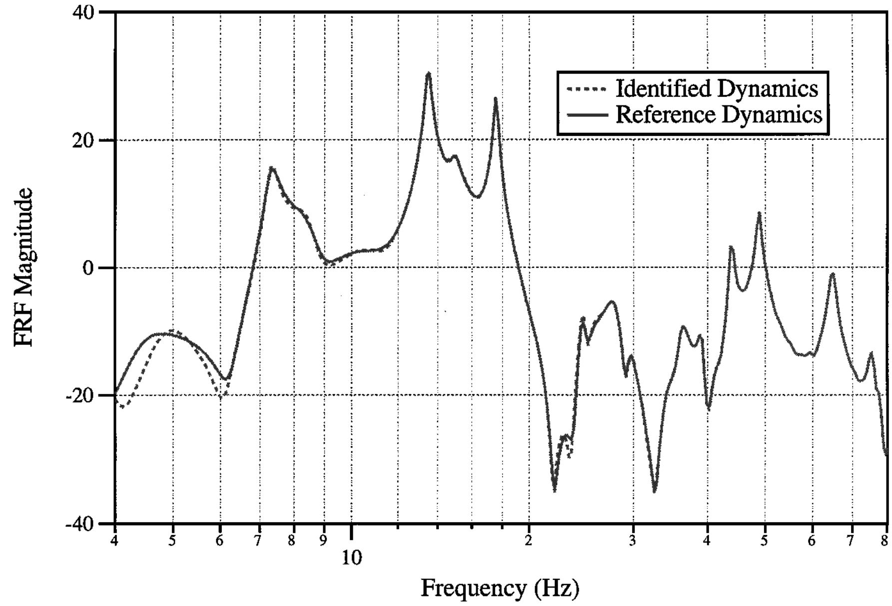

Fig. 7 Frequency responses identified with and without disturbance acting.

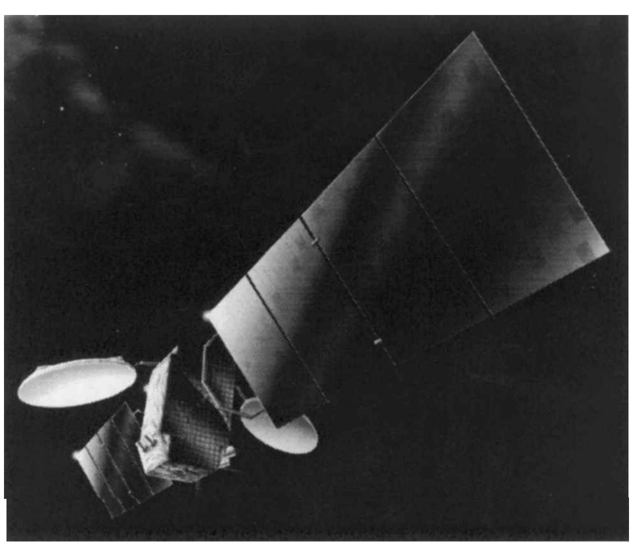

Fig. 8 Land-mobile communications satellite.

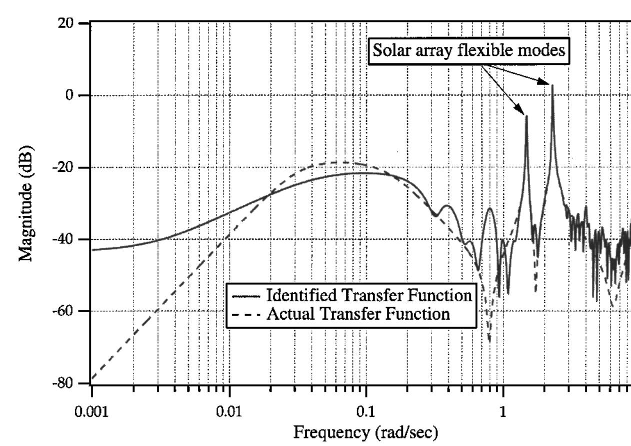

Fig. 9 Identified and actual frequency responses.

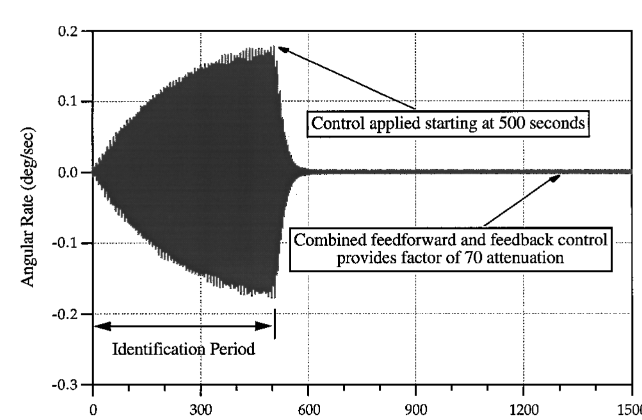

Fig. 10 Spacecraft response with feedforward and feedback control.

is essentially eliminated without inducing gain and phase errors (Fig. 7).

Spacecraft System Identification and Disturbance Rejection

Finally, a simulation study is performed to assess the feasibility of the approach on a modern spacecraft that has two flexible solar arrays attached to a rigid central core and small rigid payload reflectors as graphically depicted in Fig. 8.[17] The solar array is oriented so that both the in-plane and out-of-plane bending modes couple into the transverse body axes. Both the torque actuator and angular rate sensor are collocated on the spacecraft core. The dynamic model includes both flexible appendages with a rigid core, the rigid body controller to control the attitude dynamics, together with actuator and sensor dynamics, and a 1-s computational delay.

An "unknown" 0.028 N-m sinusoidal disturbance torque is imparted to the spacecraft body. By design this disturbance frequency coincides with a lightly damped system mode at 1.48 rad/s causing a buildup in the angular rate during the first 500 s when disturbance-corrupted data are collected for identification. During this period, an excitation input (standard deviation of 0.0034 N-m) is injected along with the rigid body control signal into the actuator. Compared to the disturbance, the excitation input has a minimal effect on the system response. This is done intentionally so that the ability to carry out system identification with minimal interference to the spacecraft system can be tested. The rate response is measured at 2 Hz (measurement noise standard deviation of 0.001 deg/s), and along with the excitation input data are used to perform system identification with $p = 60$. Although only two basis vectors are needed, 10 pairs of sines and cosines are used to confirm that adding basis vectors at harmonics not present in the disturbance does not affect the identification accuracy. Figure 9 compares the actual and identified input-output transfer functions where the primary flexible mode dynamics is accurately identified. The identification accuracy is reduced at low frequencies because of the limited data record length and the minimal excitation input, both of which are intentionally imposed here, and at higher frequencies where the excitation signal transmission is weak.

Once the identification is completed, the feedforward control is calculated. Independently, using the identified model, a predictive feedback controller is designed to enhance the system damping characteristics. The controller increases the damping of the lowest frequency flexible mode by roughly a factor of 13, from 0.3 to 3.8%. Both feedback and feedforward control are then applied simultaneously, and the system achieves steady state in about 125 s (Fig. 10), indicating that the disturbance response is attenuated by roughly a factor of 70.

Conclusions

In this paper we examine the problem of system identification in the presence of periodic or nonperiodic and possibly dominating disturbances. Available for identification are the excitation signals at the control inputs and the resultant disturbance-corrupted responses. We assume that no actual measurement of the disturbances themselves is available for identification. Several scenarios are investigated. If the actual profile of a periodic disturbance is unknown and may be quite complicated, but its period is known or can be determined, then the identification and subsequent control can be carried out without any need to resolve the disturbance into its constitutive harmonic components. If the disturbance frequencies are known only approximately, then an iterative scheme can be employed to estimate the disturbance frequencies. If the disturbance is unknown and nonperiodic but can be modeled by a set of basis functions such as orthogonal wavelets, then it is possible to use a disturbance model consisting of these basis functions to separate the effect of the unknown disturbances from the system dynamics. The proposed procedure not only produces a disturbance-free dynamics model, but it also yields results suitable for control by providing predictive information about the effect of the disturbance on the system response. Various connections between the input-output model, which is convenient for system identification, and the state-space model, which is convenient for modern control design, are developed. The connection is conveniently described in terms of an observer or Kalman filter gain from which useful insights into the identification problem are gained. Both simulation and experimental results are used to illustrate the approach. This paper focuses on the notion of modeling the disturbance effect explicitly on the system output and various ways to separate it from the system dynamics without actually knowing the disturbance input itself. For periodic disturbances the conditions under which the disturbance effect can be modeled implicitly followed by its subsequent separation from the system dynamics are investigated in a companion paper.[11] A thorough understanding of both the system identification and control aspects of the problem as well as various ways of treating it will be helpful in determining the most effective way to realize these technologies in practice.

Acknowledgment

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